Relations and Functions (संबंध एवं फलन)

Set $A = \{a, b, c\}$ = set of cities

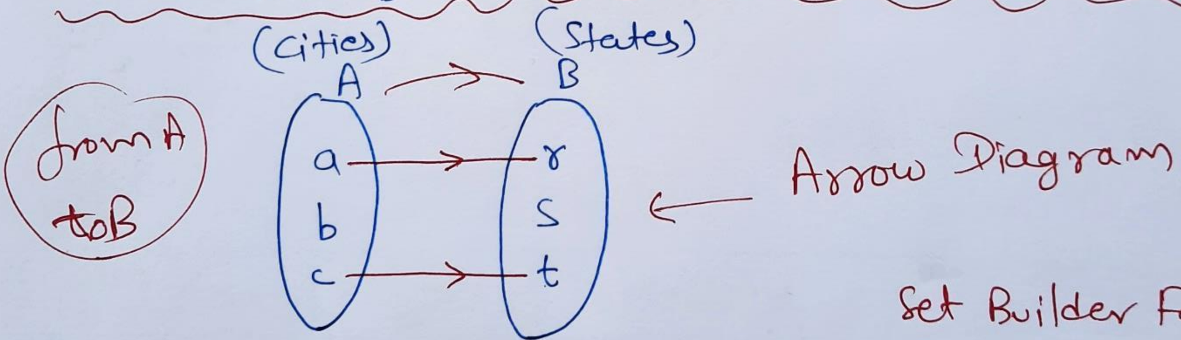
Set $B = \{r, s, t\}$ = set of states

a → Ajmer
b → Bombay
c → Chennai

r → Rajasthan
s → Sikkim
t → Tamilnadu

Cartesian Product (कार्तीय गुणन) $A \times B = \{(a,r), (a,s), (a,t), (b,r), (b,s), (b,t), (c,r), (c,s), (c,t)\}$

'set of all odered pairs from A to B'



Set Builder Form

Relation 'R₁' $= \{(x,y) : x \in A, y \in B$, City x lies in state $y\}$

Relation 'R₁' $= \{(a,r), (c,t)\}$ ← Roster Form

\# Relation is a subset of Cartesian Product A×B: from A to B $\boxed{R \subseteq A \times B}$

\# $(a,r) \in R_1$ या $a R_1 r$
$(c,t) \in R_1$ या $c R_1 t$

~~(a,s)~~
$(a,s) \notin R_1$

\# Relation A to A
⇓
Relation on 'A'
(in)

Empty Relation (रिक्त संबंध) $\phi \subseteq A \times A$

Set $A = \{1,2,3\}$

Set $B = \{100,101,102\}$

Relation $R_2 = \{(a,b) : a \in A,\ b \in B,\ \underline{a > b}\} = \{\ \} = \phi$

impossible.

Universal Relation (सार्वत्रिक संबंध) $= A \times A$

$R_3 = \{(a,b) : a \in A,\ b \in B,\ \underline{|a-b| \geq 0}\} = \begin{Bmatrix} (1,100) \\ (1,101) \\ \vdots \\ (3,102) \end{Bmatrix}$

$\{1,2,3\}$

$\{100,101,102\}$

100% Possible (Sure)

↑ All

$|1-100| = |-99| \geq 0$

$= +99 \geq 0$ True

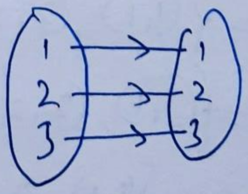
Different Types of relations on 'A'

Relation 'R' from A to A ⇒ R on 'A'

Extra

(I) Identity Relation:

$A = \{1,2,3\} \longrightarrow A = \{1,2,3\}$



$I_A = \{(1,1),(2,2),(3,3)\}$ ✓

✗ $R_1 = \{(1,1),(2,2)\}$

$R_2 = \{(1,1)(2,2)(3,3)(1,2)\}$ ✗

(II) Reflexive$^{(R)}$ Relations. (on A)

A to A

$\{1,2,3\}$ $\{1,2,3\}$

$\boxed{\text{if } (a,a) \in R \ \forall \ a \in A}$

(for every)

$A = \{1,2,3\}$

$R_1 = \{(1,1),(2,2),(3,3)\}$ Reflexive ✓ Idenitity ✓

$R_2 = \{(1,1),(2,2)\}$ Reflexive ✗ Identity ✗

$R_3 = \{(1,1),(2,2),(3,3),(1,2)\}$ Reflexive ✓ Idenityity ✗

$R_4 = \{(1,1),(2,2),(3,3),(1,2),(2,3),(3,1)\}$ Reflexive ✓ Id. ✗

(III) Symmetric Relations (सममित संबंध)

$A \longrightarrow A$

$\{1,2,3\}$ $\{1,2,3\}$

$R \Rightarrow$ if $(a,b) \in R$ then $(b,a) \in R$

$R_1 = \{(1,1),(2,2)\}$ ✓ $(1,1) \in R_1$, $(1,1) \in R_1$

$R_2 = \{(1,1),(2,2),(3,3)\}$ ✓

$R_3 = \{(1,1),(2,2),(3,3),(1,2)\}$ ✗

$R_4 = \{(1,2),(2,1),(3,2)\}$ Sym. ✗

$R_5 = \{(1,2),(2,1)\}$ Sym. ✓

(IV) Transitive Relations (संक्रामक संबंध)

if $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R$

$\Rightarrow$

Transin ✓ x

$A = \{1,2,3\} \longrightarrow A = \{1,2,3\}$

✓ $R_1 = \{(1,2),(2,3),(1,3)\}$ $(1,2) \in R_1$, $(2,3) \in R_1 \Rightarrow (1,3) \in R_1$

✓ $R_2 = \{(1,1)\}$ $(1,1) \in R_2$ $(1,1) \in R_2 \Rightarrow (1,1) \in R_2$

✓ $R_3 = \{(1,1)(2,2),(3,3)\}$

☆ $R_4 = \{(1,2)\}$ ✓

If rain then play

✓ $R_5 = \{(1,1),(2,2),(3,3),(1,2)\}$

$(1,1) \longrightarrow (1,1) \xrightarrow{\text{then}} (1,1)$

$\longrightarrow (1,2) \xrightarrow[\text{then}]{} (1,2)$

✗ $R_6 = \{(1,2),(2,3),(3,1)\}$

$(1,3) \notin R_6$

(V) Equivalence Relation (तुल्यता संबंध)

Reflexive $(a,a) \in R \ \forall a \in A$

Symmetric $(a,b) \Rightarrow (b,a)$

Transitive $(a,b),(b,c) \Rightarrow (a,c)$

e.g. $A = \{1, 2, 3\}$ $\quad$ $R : A \to A$

$R = \{(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2), (1,3) (3,1)\}$

(I) Reflexive $\quad (a, a) \in R \quad \forall\, a \in A$ ✓

(II) Symmetric ✓

$(a, b) \Rightarrow (b, a)$

(III) Transitive ✓

If $(a, b) \in R$, $(b, c) \in R$

then $(a, c) \in R$

Equivalence Relation

Exercise 1.1 [Relations and Functions]

R ~~for~~ from 'A' to 'B'

Reflexive Relations $\boxed{\text{in 'A' / on 'A'}} = \boxed{\text{from A to A}}$

$(a,a) \in R$ for every $a \in A$.

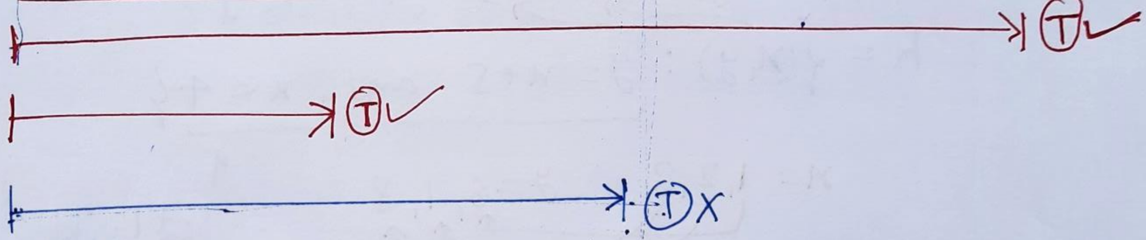
Equivalence Relation

Symmetric Relations

if $(a,b) \in R$ then $(b,a) \in R$.

Transitive Relations

If $(a,b) \in R$ and $(b,c) \in R$ then $\underline{(a,c) \in R}$.



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Exercise 1.1

$\boxed{Q.1}$ Reflexive / Symmetric / Transitive

(i) $A = \{1,2,3,---13,14\}$ (in A) $x \in A$, $y \in A$

$R = \{(x,y) : 3x - y = 0\}$

Reflexive, for (x,x) $3x - x = 0 \Rightarrow 2x = 0 \Rightarrow \boxed{x = 0 \notin A}$

$(0,0)$ $\quad x \notin A$

Not reflexive.

Symmetric If $\underbrace{(x,y) \in R}_{3x-y=0}$ then $\underbrace{(y,x) \notin R}_{3y-x=0}$ $\quad 3x-y=0 \not\Rightarrow 3y-x=0$

Not Sym.

Transitive,

If $(x,y) \in R$ and $(y,z) \in R$ then $(x,z) \notin R$

$\downarrow$ $\downarrow$ $\downarrow$

$3x-y=0$ —① $3y-z=0$ —② $\boxed{3x-z=0}$??

$3x=y$ $\Rightarrow 3(3x)-z=0$

$\boxed{9x-z=0}$

Not Transitive

(ii) Relation in N ← (set of natural no.)

(from N to N) $\{1,2,3\ldots\}$

$R=\{(x,y) : \underbrace{y=x+5 \text{ and } x<4}\}$

Set Builder Form

$x=1,2,3$ $y=6,7,8$

$R=\{(1,6),(2,7),(3,8)\}$

Reflexive $(1,1)\ (2,2)\ (3,3)\ (4,4)\ldots$ ×

Not reflexive

Symmetric $(1,6) \in R$ but $(6,1) \notin R$

$\therefore$ Not sym.

Transitive,

$(1,6) \in R$

$(2,7) \in R$

$(3,8) \in R$

only

Transitive,

(iii) Relation in the set $A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(x, y) : y \text{ is divisible by } x\}$

Reflexive. $(x, x) \in R \ \forall x \in A$

$\downarrow$

x is divisible by x $\forall x \in A$

(True)

Yes. Reflexive ✓

Symmetric : If $(x, y) \in R$ then $(y, x) \in R$.

y is divisible by x

6 —||— 2 ✓

'x' is divisible by 'y'

2 —||— 6 ✗

Not sym.

Transitive. If $(x, y) \in R$ and $(y, z) \in R$ then $(x, z) \in R$

y is divisible by x

$y = \boxed{kx}$

z is divisible by y

$z = m.y$

$\Rightarrow z = mk.x$

z is divi. by x

$z = \boxed{mk}.x$

Yes Transitive.

(iv) Relation in set Z ← (all integers)

$\{\ldots -2, -1, 0, 1, 2, 3 \ldots\}$

$R = \{(x, y) : x - y \text{ is an integer}\}$

Reflexive. $\underbrace{(x, x) \in R}_{\downarrow}$ $\forall$ $x \in Z$ ↳ for every

✓ Yes

$\underbrace{x - x}_{0}$ is an integer $\forall\ x \in Z$

0 ✓ True

Symmetric If $(x, y) \in R$ then $(y, x) \in R$

Yes

$(x - y)$ is an integer ; $(y - x)$ is an integer

$(x - y) = I$

$\Rightarrow -(y - x) = I$

$\Rightarrow (y - x) = -I$

Transitive. If $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$

yes

$\underbrace{(a - b) \text{ is an integer}}$ & $\underbrace{(b - c) \text{ is an integer}}$; $\underbrace{(a - c) \text{ is an integer}}$

$$a - \cancel{b} = I_1$$

$$\cancel{b} - c = I_2$$

$$+$$

$$\underbrace{a - c}\ = I_1 + I_2 = \text{integer}$$

✓

(v) relation R in set 'A' of human beings in a town.

ⓐ $R = \{(x,y) : x \text{ and } y \text{ work at the same place}\}$

ⓑ $R = \{(x,y) : x \text{ and } y \text{ live in the same locality}\}$

Reflexive	Symmetric	Transitive
$(x,x) \in R$ ✓	$(x,y) \in R \Rightarrow (y,x) \in R$ ✓ ✓	$(x,y) \in R, (y,z) \in R$ $\Rightarrow$ $(x,z) \in R$ ✓

ⓒ $R = \{(x,y) : x \text{ is exactly } 7\text{cm taller than } y\}$

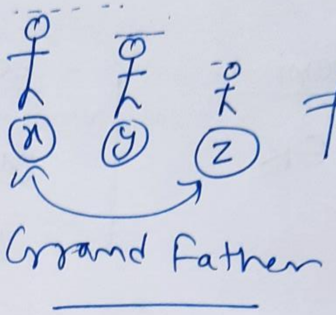
Reflexive	Sym	Transitive
$(x,x) \notin R$ Not ✓	$(x,y) \in R$ $\Rightarrow$ $(y,x) \notin R$ × Not ✓	$(x,y) \in R$ & $(y,z) \in R$ $\Rightarrow$ $(x,z) \notin R$ $x = z+7$ × $x = y+7$ ① $y = z+7$ ② $x = (z+7)+7$ $x = z+14$ Not ✓

ⓓ $R = \{(x,y) : x \text{ is wife of } y\}$

Reflexive	Sym.	Transitive
$(x,x) \in R$ × Not	$(x,y) \in R \Rightarrow (y,x) \in R$ W H ✓ W H ? ×	$(x,y) \in R, (y,z) \notin R \Rightarrow (x,z) \in R$ W H ~~W H~~ Transitive

ⓔ $R = \{(x,y) : x \text{ is father of } y\}$

Reflexive	Sym.	Transitive
$(x,x) \in R$	$\underline{(x,y) \in R} \Rightarrow \underline{(y,x) \in R}$	$\underbrace{(x,y) \in R, (y,z) \in R}$ $\Rightarrow \underbrace{(x,z) \notin R}$
X	X	Father



$\not\Rightarrow$

X

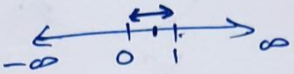
Exercise 1.1 (Relations and Functions)

Q.2 Relation in the set 'R' ← (real numbers)

$R = \{(a,b) : a \leq b^2\}$ → Reflexive ✗, Symmetric ✗, Transitive ✗ } Prove

Reflexive

real no.



$(x,x) \in R \ \forall \ x \in R$

relation

for every

$\Rightarrow x \leq x^2$

$x = 0.5 = \frac{1}{2}$

-1, -2

1, 2, 3, 4, ... ✓

0 ✓

$\frac{1}{2} \nleq \left(\frac{1}{2}\right)^2$

$\frac{1}{2} \nleq \frac{1}{4}$

$0.5 > 0.25$

This is not true for $x \in (0,1)$

Not reflexive.

Symmetric If $(x,y) \in R$ then $(y,x) \in R$

$x \leq y^2 \xrightarrow{\times} y \leq x^2$

$x = 1, \ y = 2$

$1 \leq 4$ ✓

$2 \leq 1$ ✗

Not Sym.

Transitive If $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \notin R$

$a \leq b^2$ ✓ $b \leq c^2$ ✓ } $a \leq c^2$

Not ✓

$7 \leq 25$ $a = 7, b = -5$ $c = 1$ $\boxed{7 \leq 1}$ ✗

$-5 \leq 1$ ✓

Q.3 Relation (R) in the set $A = \{1, 2, 3, 4, 5, 6\}$

$R = \{(a, b) : b = a+1\}$ (a, b)

Reflexive.

$(a,a) \in R \ \forall a \in A$

$a = a+1$

$\Rightarrow 0 = 1$

False

Not Reflexive

Symmetric

$(a,b) \in R \Rightarrow (b,a) \in R$

$b = a+1 \qquad a = b+1$

✗

Not

Transitive

$(a,b) \in R,\ (b,c) \in R$

$\Rightarrow (a,c) \notin R$

$b = a+1$ — ①

$c = \cancel{b}+1$ — ②

+

$c = a+2$

$(a,c) \rightarrow c = a+1$

Not

Q.4 Relation 'R' in set 'R' ← Real no.

$R = \{(a,b) : a \le b\}$

- Reflexive ✓
- Transitive ✓
- Sym. ✗

Reflexive.

$(a,a) \in R$

$a \le a$

✓

✓

Sym.

$(a,b) \in R \Rightarrow (b,a) \in R$

$a \le b \qquad b \le a$

$2 \le 5 \qquad 5 \le 2$ ✗

Not Sym.

Transitive.

$(a,b) \in R,\ (b,c) \in R$

$\boxed{a \le b} \qquad b \le c$

a b c

$\Downarrow$

$(a,c) \in R$

$a \le c$ ✓

Transitive ✓

Q.5 relation R in $\mathbb{R}$ → real no.

$R = \{(a,b) : a \le b^3\}$

Reflexive

(a,a) ∈ R ∀a∈R

$a \nleq a^3$

$a = \frac{1}{2}$

$\frac{1}{2} \nleq \left(\frac{1}{2}\right)^3$

$\frac{1}{2} \nleq \frac{1}{8}$

↓ ↓

$0.5 > 0.125$

Not

Transitive

(a,b) ∈ R , (b,c) ∈ R

$a \le b^3 \leftarrow \Downarrow \; b \le c^3$

$(a,c) \notin R$

$a \nleq c^3$

$a \le (b)^3 \le (c^3)^3$

$a \le c^9$

Not transitive

Symmetric

(a,b) ∈ R ⇒ (b,a) ∈ R

$a \le b^3 \rightarrow b \le a^3$ ×

a = 1

b = 2

Not Sym.

Q.6 Relation R in the set {1,2,3}

$R = \{(1,2),(2,1)\}$ → Sym. ✓
→ Reflexive ×
→ Trans. ×

Reflexive

{(1,1) (2,2), (3,3)}

×

Sym.

(1,2) ∈ R then (2,1) ∈ R

✓

Transitive, $(a,b),(b,c) \Rightarrow (a,c)$

$(1,2)\;(2,1) \Rightarrow (1,1) \notin R$

Not transitive

$\boxed{Q.7}$ in set 'A' of all books in a library.

$R = \{(x, y) : x$ and y have same no. of pages$\}$

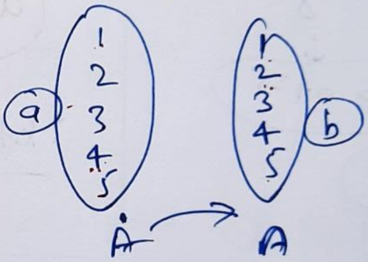
↙

Equivalence

- Reflexive → $(x, x) \rightarrow$ ✓
- Symmetric → $(x, y) \in R \Rightarrow (y, x) \in R$ ✓
- Transitive → $(x, y) \in R$, $(y, z) \in R \Rightarrow (x, z) \in R$ ✓

$\boxed{Q.8}$ Relation in set $A = \{1, 2, 3, 4, 5\}$

$R = \{(a, b) : |a - b|$ is even$\}$



$R = \{(1,1), (1,3), (1,5),$
$(2,2), (2,4),$
$(3,1), (3,3), (3,5),$
$(4,2), (4,4),$
$(5,1), (5,3), (5,5)\}$

$\{1, 3, 5\}$

$|odd - odd| = even$

$|even - even| = even$

Reflexive

$(a, a) \in R \ \forall a \in A$

✓

Symmetric

$(a, b) \Rightarrow (b, a)$

✓

Transitive

$(a, b), (b, c) \Rightarrow (a, c)$

✓

Q.9 Relation in the set $A = \{x \in \mathbf{Z} : 0 \le x \le 12\}$

$A = \{0, 1, 2, 3 \dots 12\}$

(i) $R = \{(a, b) : |a-b| \text{ is a multiple of } 4\}$

(ii) $R = \{(a, b) : a = b\}$

(i) $R = \{(a, b) : |a-b| \text{ is a multiple of } 4\}$

Reflexive. $(a, a) \in R \ \forall \ a \in A$

✓ $\Rightarrow |a-a| = 0$ is a multiple of 4 ✓

Sym. $(a, b) \in R \Rightarrow (b, a) \in R$

✓ $|a-b|$ is a multiple of 4 $\qquad$ $|b-a|$ is a multiple of 4

True.

Transitive. ✓ $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$

$|a-b| = 4k \qquad |b-c| = 4m \qquad |a-c| = 4n$

$a-b = 4k \qquad b-c = 4m$

$\oplus$

$a-c = 4k + 4m$

$a-c = 4(k+m)$

$|a-c| = 4|k+m|$

elements related to ①

$(a,b) \in R$

$\boxed{a=1}$ $\boxed{b=?}$

(i) $|a-b|$ is a multiple of 4

$0, 4, 8, 12, \ldots$

$|1-b|$ is a multiple of 4

$b = 1, 5, 9$

(ii) $\boxed{a=b}$

(a,a) —

$(a,b)\ (b,a)$ —

$(a,b)\ (b,c)\ (a,c)$ —

$\boxed{a=1}$ $(a,b) \in R$

$1 = b$ $\quad$ 1

$\boxed{a=b}$

Exercise 1.1 (Relations and Functions)

$\boxed{Q11 \text{ to } Q16}$

Q.11 Relation in the set A of points in a plane.

$R = \{(P, Q)$: distance of the point P from origin is same as the distance of the point Q from origin$\}$

$\downarrow$

$OP = OQ$

Reflexive

$(P, P) \in R \ \forall \ P \in A$

$\downarrow$

origin = 'O' $(0, 0)$

$OP = OP$ True. ✓

Reflexive ✓

Symmetric

$\underbrace{(P, Q) \in R}_{\swarrow} \Rightarrow \underbrace{(Q, P) \in R}_{\uparrow}$

$OP = OQ \longrightarrow OQ = OP$

✓

Transitive

$(P, Q) \in R, \ (Q, S) \in R$

$OP = OQ$ ① $OQ = OS$ ②

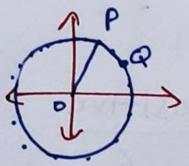
$\Downarrow$

$(P, S) \in R$

$\boxed{OP = OS}$

✓

Equivalence ✓



Sides

$T_1 \rightarrow 3, 4, 5$

$T_2 \rightarrow 5, 12, 13$

$T_3 \rightarrow 6, 8, 10$

$T_1 \sim T_3$

Q.12 Relation in the set A of all triangles.

$R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2\} \rightarrow T_1 \sim T_2$

Reflexive

$T_1 \sim T_1$ ✓

Symmetric

$\underline{T_1 \sim T_2} \Rightarrow \underline{T_2 \sim T_1}$ ✓

Transitive

$T_1 \sim T_2, \ T_2 \sim T_3 \Rightarrow \boxed{T_1 \sim T_3}$ ✓

Q.13 Relation in the set A of all polygons

$R = \{(P_1, P_2) : P_1 \text{ and } P_2 \text{ have same no. of sides}\}$

$\boxed{n(P_1) = n(P_2)}$

let: $n(P)$ denotes no. of sides of 'P'

Reflexive.

$(P_1, P_1) \in R \ \forall \ P_1 \in A$

$n(P_1) = n(P_1)$

✓

Sym.

$(P_1, P_2) \in R \Rightarrow (P_2, P_1) \in R$

$n(P_1) = n(P_2) \quad n(P_2) = n(P_1)$

✓

Transitive $(P_1, P_2) \in R, \ (P_2, P_3) \in R \Rightarrow (P_1, P_3) \in R$

$n(P_1) = n(P_2)$ —①, $\quad n(P_2) = n(P_3)$ —②

$\therefore n(P_1) = n(P_3)$

✓

Q.14 Relation in set L of all lines in XY plane.

$R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$

$\boxed{L_1 \parallel L_2}$

Reflexive

$(L_1, L_1) \in R$

$\boxed{L_1 \parallel L_1}$

✓

Sym.

$(L_1, L_2) \in R \Rightarrow (L_2, L_1) \in R$

$\downarrow \qquad \downarrow$

$L_1 \parallel L_2 \qquad L_2 \parallel L_1$

✓

Transitive

$(L_1, L_2), (L_2, L_3) \Rightarrow (L_1, L_3)$

$\downarrow \qquad \downarrow \qquad \downarrow$

$L_1 \parallel L_2 \qquad L_2 \parallel L_3 \qquad L_1 \parallel L_3$

✓

$\xleftrightarrow{m} L_1$

$\xleftrightarrow{m} L_2$

$\xleftrightarrow{m} L_3$

$\boxed{y = \underset{\text{Slope}}{m}x + c}$

Equivalence ✓

$\boxed{y = 2x + 4} \xrightarrow[\parallel]{} \boxed{y = 2x + c} \quad c \in R$

Q.15 Relation in the set $\{1,2,3,4\} = A$

$$R = \{(1,2), \underline{(2,2)}, \underline{(1,1)}, \underline{(4,4)}, (1,3), \underline{(3,3)}, (3,2)\}$$

Reflexive	Symmetric	Transitive
$(a,a) \in R$ $\forall\, a \in A$ ✓ ↓ Option (B)	Here $(1,2) \in R$ but $(2,1) \notin R$ ✗	$(1,2) \in R \quad (2,2) \in R$ $(1,2) \in R$ ✓ $\boxed{(a,b), (b,c) \Rightarrow (a,c)}$ ✓

Q.16 Relation in the set $\underline{N}$ ✓

$$R = \{(a,b) : \boxed{a = b-2},\ b > 6\}$$

Ⓐ $(2,4) \in R$ ✗ $4 < 6$

Ⓑ $(3,8) \in R$ $3 \neq 8-2$

Ⓒ $(6,8) \in R$ $6 = 8-2$ ✓

Ⓓ $(8,7) \in R$ $8 \neq 7-2$

Option Ⓒ

→ One-one Function (Injective Fn.) (एकैकी फलन)

→ many-one Function (बहुएकैकी फलन)

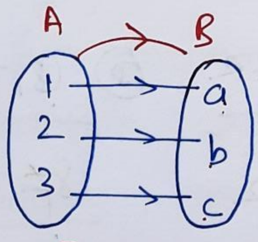
→ Onto Function (Surjective Fn.) (आच्छादक फलन)

→ Into Function (अन्तर्क्षेपी फलन)

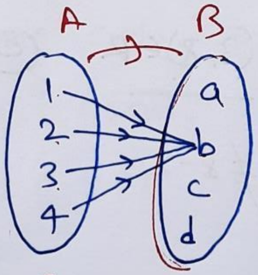
⇒ One One Onto Fn. (Bijective Fn.)

Function (फलन)

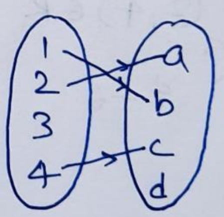
Function is a special type of relation from a non empty set A to another non empty set B such that each element of set A has unique image in set B.



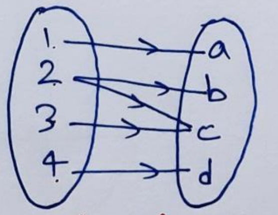
Function



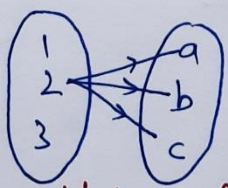
Function.



Not a function



Not a function



Not a func.

image of '1' = a
Preimage of 'a' = 1

function $f : A \to B$

Domain — input (x)

Codomain

Range = Set of images

output (y = f(x))

Range ⊆ Codomain

One One Functions (Injective Fn.) (एकैकी फलन)

$f: X \to Y$

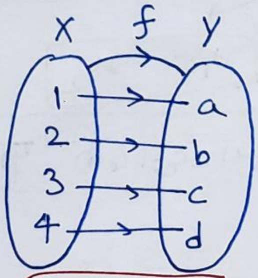
images of distinct elements of X under 'f' are distinct

$x_1, x_2 \in X$

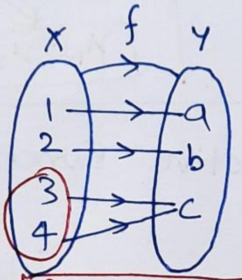
$\boxed{f(x_1) = f(x_2) \Rightarrow x_1 = x_2}$ only

$\boxed{x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)}$

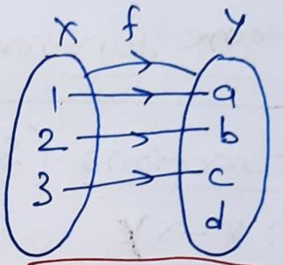
Many-one Functions : which are not one-one.



one-one



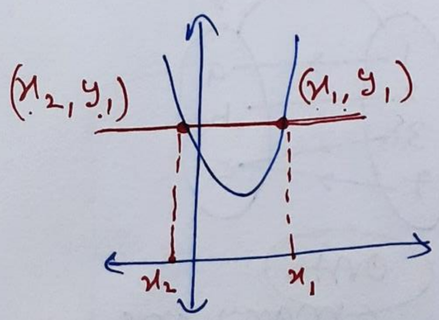
many-one

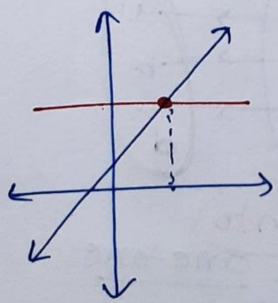


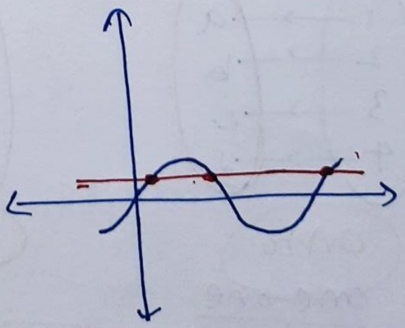
One-one

Horizontal line Test : (for graphs)

If a Horizontal line cuts the graph of a function at atmost (ज्यादा से ज्यादा) one point, then this graph represents a one-one function.



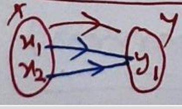




many-one

one-one

many-one Fn.



e.g. Check injectivity (one-one / many-one)

(i) $f: \textcircled{R} \to R$

$f(x) = 3x+7$

let $x_1, x_2 \in R$

$f(x_1) = f(x_2)$

$\Rightarrow 3x_1 + \cancel{7} = 3x_2 + \cancel{7}$

$\Rightarrow \boxed{x_1 = x_2}$ only.

one-one function

(ii) $f: \textcircled{R} \to R$

$f(x) = x^2$

let $x_1, x_2 \in \textcircled{R}$

$f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow \boxed{x_1 = \pm x_2}$ many-one

$\boxed{x_1 = x_2}$ $\boxed{x_1 = -x_2}$

Onto Functions (Surjective Function) (आच्छादक फलन)

(पर) $f: X \to Y$

if every element of Y, is the image of some element of X under f.

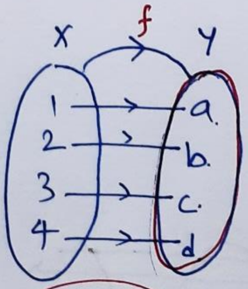
$\boxed{\text{Range} = \text{Codomain}(Y)}$

Connected in Y

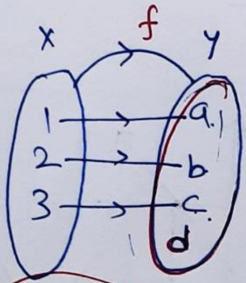
(में)

Into Functions: which are not onto.

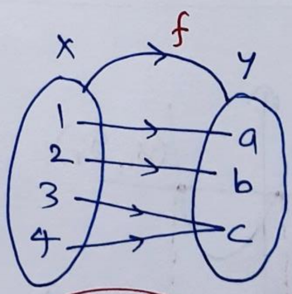
eg.



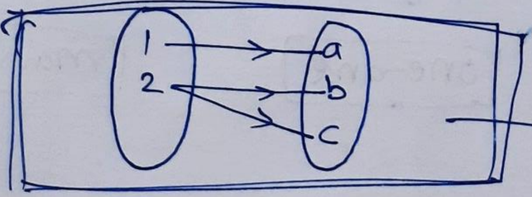
Onto
~~one-one~~



Into
one-one



Onto
many-one



→ Comment

Bijective Functions (one-one Onto Functions)

which are one one & onto both.

$f(x_1) = f(x_2) \rightarrow x_1 = x_2$

Range = codomain

e.g. Check Surjectivity (Hence Bijectivity)

onto / into

(i) $f(x) = 3x+7$

$f: R \rightarrow R$

Codomain

$x \in R$ ← Domain

$3x+7 \in R$

$f(x) \in R$ ← Range.

output Range = R = Codomain

onto ✓

one-one — Bijective

(ii) $f(x) = x^2$

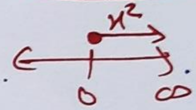
$f: R \rightarrow R$ ← Codomain.

Range.

$x \in R$

$x^2 \geq 0$ Square ≥ 0

$x^2 \in [0, \infty)$



$f(x) \in [0, \infty)$ ← Range.

output

Range = $[0, \infty) \neq R$ $(-\infty, \infty)$

into

many-one

Tip ☆ Questions with $f: N \rightarrow N$ — Draw Arrow Diagram

1, 2, 3...

☆ Questions with $f: R \rightarrow R$

Apply Proper methods

$f(x_1) = f(x_2)$ (one-one) ↘ $x_1 = x_2$

Range = Codomain (onto)

Exercise 1.2 [Relations and Functions]

$\boxed{Q.1}$ $f : \overset{\text{Domain}}{\overline{R_*}} \rightarrow \overset{\text{Codomain}}{\overline{R_*}}$ defined by $\underbrace{f(x) = \frac{1}{x}}$

$R - \{0\} = (-\infty, 0) \cup (0, \infty)$

One-one $\overset{\text{let}}{=} f(x_1) = f(x_2) \xrightarrow{\text{Prove}} \boxed{x_1 = x_2}$ only

$\Rightarrow \frac{1}{x_1} = \frac{1}{x_2}$

$\Rightarrow x_1 = x_2$ only

$\therefore f \rightarrow$ one-one ✓

onto $\boxed{\text{Range} = \text{codomain} \; (R_*)}$ Prove

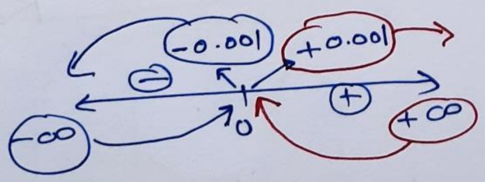
$\frac{1}{0} = \infty$

$\frac{1}{\infty} = 0$

really output

$x \rightarrow$ input

$\underbrace{y, f(x)} \rightarrow$ output = Range



onto proved ✓

Range.

$f(x) = \frac{1}{x}$

Domain (input)

$x \in (-\infty, 0) \cup (0, \infty)$

$\frac{1}{x} \in (-\infty, 0) \cup (0, \infty)$

$f(x) \in (-\infty, 0) \cup (0, \infty)$

Output

Range $= (-\infty, 0) \cup (0, \infty)$

$= (-\infty, \infty) - \{0\}$

$= R - \{0\}$

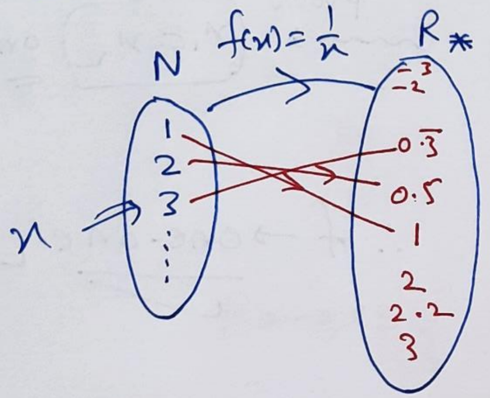
$= R_* =$ Codomain

Original function $f: R_* \to R_*$, $f(x) = \frac{1}{x}$ $\boxed{\text{one-one onto}}$

Next Case $f: N \to R_*$, $f(x) = \frac{1}{x}$ $\begin{matrix} \to ? \\ \to ? \end{matrix}$

$N \downarrow \{1, 2, 3, \ldots\}$

Arrow Diagram.



$f(1) = \frac{1}{1} = 1$

$f(2) = \frac{1}{2} = 0.5$

$f(3) = \frac{1}{3} = 0.\overline{3}$

$f(4) = \frac{1}{4} = 0.25$

one-one

✓

onto → ✗

R_* के सारे elements connected

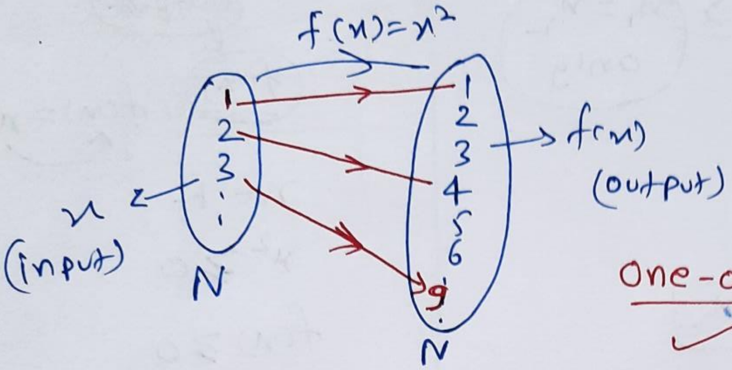
into

Range ≠ Codomain

R_*

Q.2 Check injectivity & Surjectivity
(one-one) (onto)
(manyone) (into)

(i) $f: N \to \overset{\text{Codomain}}{\textcircled{N}}$ given by $f(x) = x^2$



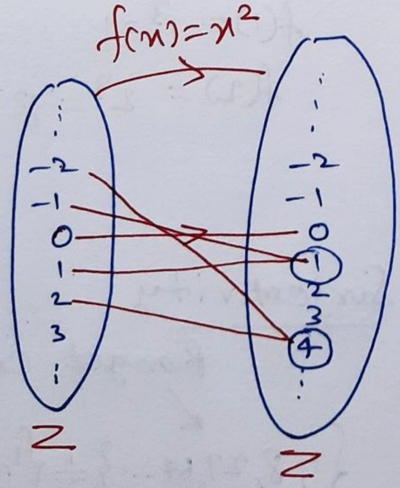
$f(1) = 1^2 = 1$
$f(2) = 2^2 = 4$

One-one ✓ | onto X
Range ≠ Codomain

Range = $\{1, 4, 9, 16, \ldots\} \neq \{1, 2, 3 \ldots\}$ (N)

one-one & into

(ii) $f: Z \to \textcircled{Z}$ given by $f(x) = x^2$
Z ↓ integers



$f(0) = 0^2 = 0$
$f(-1) = (-1)^2 = 1$
$f(-2) = (-2)^2 = 4$
$f(1) = 1^2 = 1$
$f(2) = 2^2 = 4$

~~Ans~~ Injectivity
many-one ✓

Surjectivity
Range ≠ Codomain
$\{0, 1, 4, 9, 16, \ldots\} \neq Z$
x^2
into

(iii) $f: R \to \textcircled{R}$ given by $f(x) = x^2$

$(-\infty, \infty)$

One-one $x_1, x_2 \in R$??

$f(x_1) = f(x_2) \to \boxed{x_1 = x_2 \text{ only}}$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2$

$\boxed{x_1 = x_2}$ या $\boxed{x_1 = -x_2}$

many one

onto Range = Codomain $\textcircled{R}$

output

f(x)

$f(x) = x^2$

$x \in R$

$x^2 \geq 0$

$f(x) \geq 0$

$f(x) \in [0, \infty)$

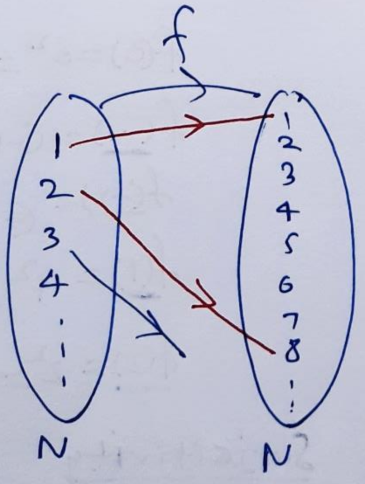
Range $= [0, \infty)$

Codomain $= R \,(-\infty, \infty)$

Range ≠ Codomain

Into

(iv) $f: N \to \textcircled{N}$ given by $f(x) = x^3$



$f(x) = x^3$

$f(1) = 1^3 = 1$

$f(2) = 2^3 = 8$

One-one

✓

Surjectivity

Range ≠ Codomain

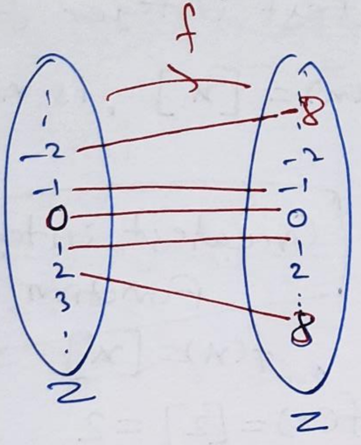
N

$\{1, 8, 27, 64 \ldots\} \neq \{1, 2, 3 \ldots\}$

into

(V) $f : Z \to Z$ given by $f(x) = x^3$

$\{-2, -1, 0, 1, 2, 3 - -\}$



$f(x) = x3$

$f(0) = 0$

$f(1) = 1$

$f(2) = 8$

$f(3) = 27$

$f(-1) = -1$

$f(-2) = -8$

$f(-3) = -27$

one-one ✓

onto ✗ into ✓

Range ≠ Codomain

$\{ -- \underline{27, 8\ 1, 0, -1, -8 \ldots} \} \neq (Z)$

Exercise 1.2 (Relations and Functions)

$\boxed{Q.3}$ Prove that the greatest integer function $f: R \to \textcircled{R}$, given by $f(x) = [x]$, is neither one-one nor onto.

one-one $\quad f: \underset{\underset{\text{input}}{\textcircled{x}}}{\underset{\uparrow}{R}} \to R$

$f(x) = [x]$

$x_1 = 8.2$

$f(x_1) = f(8.2) = [8.2] = 8$

$x_2 = 8.5$

$f(x_2) = f(8.5) = [8.5] = 8$

$x_1 \neq x_2$

but $f(x_1) = f(x_2)$

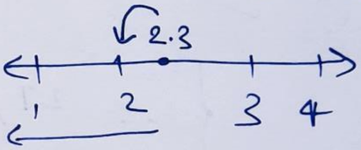
$\boxed{\text{not one-one}}$

Greatest integer Function

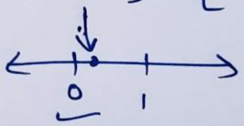
$f(x) = [x]$

$f(2) = [2] = 2$

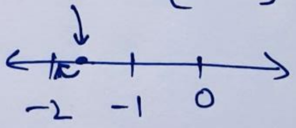
$f(2.3) = [2.3] = 2$



$f(0.02) = [0.02] = 0$



$f(-1.7) = [-1.7] = -2$



onto / into

$\underset{\text{actual output}}{\underset{\downarrow}{\text{Range}}} = \underset{\textcircled{R}}{\text{Codomain}}$

$f(x) = [x] = \text{Integer} = Z$

$\text{Range} = Z = \{\ldots -2, -1, 0, 1, 2, \ldots\}$

$\text{Range} = Z$ $\neq$
$\text{Codomain} = R$

into

$\boxed{Q.4}$ $f: R \to \textcircled{R}$ $f(x) = |x|$

$\boxed{\text{neither one-one nor onto}}$

<u>one-one.</u>

$f(x_1) = f(x_2) \Rightarrow \boxed{x_1 = x_2}$ only ??

$\Rightarrow |x_1| = |x_2|$

$\Rightarrow x_1 = \pm x_2$ → $x_1 = x_2$, $x_1 = -x_2$

$2 = 2$

$|2| = |2|$ ✓

$|-2| = |2|$ ✓

$|-2| = |-2|$

Not one-one

$f(x) = |x|$

$\underline{x_1 = 2} \neq \underline{x_2 = -2}$

$f(x_1) = f(x_2)$

$\downarrow \quad \downarrow$

$2 = 2$

<u>onto</u>

Range = Codomain (R)

$\downarrow$

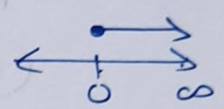
output

$\downarrow$

$f(x) = |x|$

$|x| \geq 0$

$\infty > \underline{f(x)} \geq 0$



Range $= [0, \infty)$

$\neq$

Codomain $= R = (-\infty, \infty)$

<u>Into</u>

<u>not onto</u> ✓

$\boxed{Q.5}$ $f: R \to \textcircled{R}$ $f(x) = \begin{cases} 1 & ; x > 0 \\ 0 & ; x = 0 \\ -1 & , x < 0 \end{cases}$

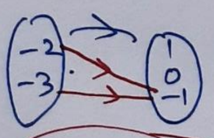
(<u>Signum</u>)

neither one one nor onto

<u>Not one-one.</u>

$f(2) = 1$, $f(3) = 1$

$f(-2) = -1$, $f(-3) = -1$



not one-one

many-one

<u>not onto</u> ✓

Range $\neq$ Codomain (R)

$\downarrow$

Output

$\downarrow$

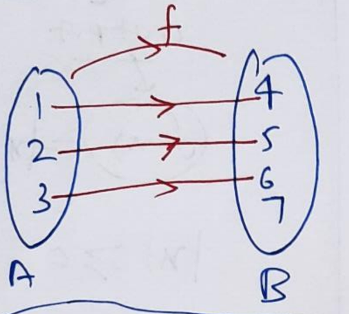
$\{1, 0, -1\}$ ~~ont~~

not onto

$\boxed{Q.6.}$ $A = \{1, 2, 3\}$ $B = \{4, 5, 6, 7\}$

$f = \{(1,4), (2,5), (3,6)\}$ $f : \underline{A} \to \underline{B}$

$\boxed{\text{Show that } f \text{ is one-one}}$



one - one

$\boxed{Q.7}$ (i) $\underbrace{f : R \to R}$ defined by $f(x) = 3 - 4x$

One-one.

Let $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ only

$f(x_1) = f(x_2)$

$\Rightarrow \cancel{3} - 4x_1 = \cancel{3} - 4x_2$

$\Rightarrow \boxed{x_1 = x_2}$ only

$\boxed{\text{One-one}}$

Onto $\boxed{\text{Range} = \text{Codomain}}$

Output

$(f(x))$

R

$x \in R$

$3 - 4x \in R$

$f(x) \in R$

Range $= R =$ codomain

✓

onto

Bijective

(ii) $f: R \to \textcircled{R}$ defined by $f(x) = 1+x^2$

$$\underline{f(x) = 1+x^2}$$

one-one

let $f(x_1) = f(x_2)$

$\Rightarrow \cancel{1}+x_1^2 = \cancel{1}+x_2^2$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow \boxed{x_1 = \pm x_2}$

$(x_1 = x_2)$ या $\boxed{x_1 = -x_2}$

not one-one

onto Range = Codomain

??

$\textcircled{R}$

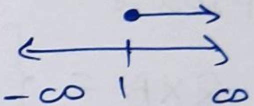
$(-\infty, \infty)$

$x \in R$

$x^2 \geq 0$

$\underbrace{1+x^2} \geq \underbrace{1+0}$

$f(x) \geq 1$



$f(x) \in [1, \infty)$

↑

Range

Range ≠ Codomain

← not onto

not ~~Bij~~
Bijective.

Exercise 1.2 (Relations and Functions)

Q.8 Let A and B be sets. Show that $f : A \times B \to \boxed{B \times A}$ such that $f(\underline{a,b}) = (\underline{b,a})$ is bijective function. one-one & onto

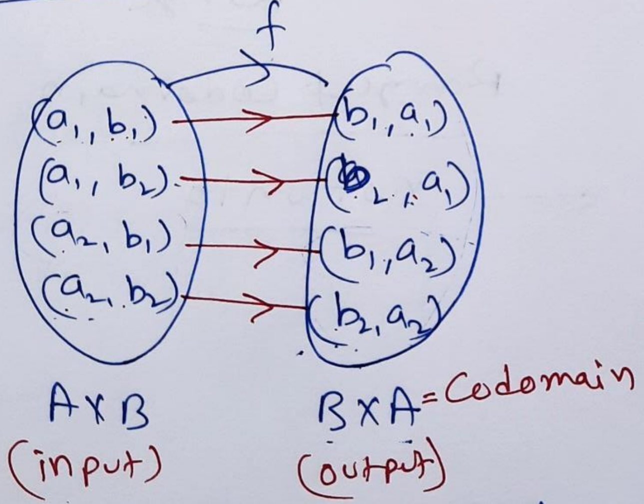
Ans. Let $A = \{a_1, a_2\}$ $B = \{b_1, b_2\}$

Cartesian Product

$$A \times B = \{(a_1, b_1), (a_1, b_2), (a_2, b_1), (a_2, b_2)\}$$

$$B \times A = \{(b_1, a_1), (b_2, a_1), (b_1, a_2), (b_2, a_2)\}$$

Arrow Diagram



$$f(a,b) = (b,a)$$

$$f(x,y) = (y,x)$$

$$f(1,-3) = (-3,1)$$

$$f(\underline{a_1,b_1}) = (\underline{b_1,a_1})$$

One-one

✓

✓

onto Range = Codomain

↓ ↓

$B \times A = B \times A$

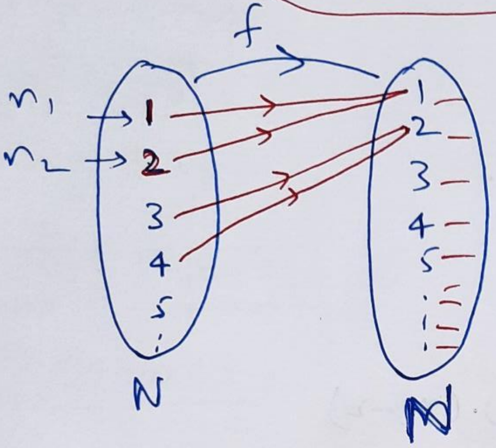
✓

✓

Bijective Fn.

Q.9 $f: N \to N$ be defined by

$$\boxed{f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}}; \quad n \in N$$



$f(1) = \frac{1+1}{2} = 1$

$f(2) = \frac{2}{2} = 1$

$f(3) = \frac{3+1}{2} = 2$

$f(4) = \frac{4}{2} = 2$

⋮

One-one → Not

(many-one) ✓

$n_1 = 1, n_2 = 2$

$n_1 \neq n_2$ ←

$f(n_1) = f(n_2)$

$f(1) = f(2)$

$\downarrow \quad \downarrow$

$1 \quad 1$

Onto / Into ✓

↓

Range = Codomain

↓ ↓

Output N ✓

$\{1, 2, 3, \ldots\}$

⇓

N ✓

Onto ✓

↘ ↙

not one one onto

↓ ↓

Not Bijective

$\boxed{Q.10}$

$$f:\underset{x}{\overset{R-\{3\}}{A}} \longrightarrow \boxed{\overset{R-\{1\}}{B}} \text{ defined by } f(x)=\frac{x-2}{x-3}$$

one-one

let $f(x_1)=f(x_2) \Rightarrow \overset{\text{only}}{\boxed{x_1=x_2}}$??

$\Downarrow$

let $f(x_1)=f(x_2)$

$$\Rightarrow \frac{x_1-2}{x_1-3}=\frac{x_2-2}{x_2-3}$$

$$\Rightarrow (x_1-2)\cdot(x_2-3)=(x_1-3)\cdot(x_2-2)$$

$$\Rightarrow \cancel{x_1x_2}-3x_1-2x_2+\cancel{6}=\cancel{x_1x_2}-2x_1-3x_2+\cancel{6}$$

$$\Rightarrow -3x_1+2x_1=2x_2-3x_2$$

$$\Rightarrow \cancel{-}x_1=\cancel{-}x_2 \Rightarrow \boxed{x_1=x_2} \text{ only}$$

$\therefore f(x) \rightarrow$ one-one ✓

---

onto $\boxed{\text{Range} = \text{Codomain}}$

output $f(x)=y$ $\quad$ $\boxed{B=R-\{1\}}$

$y=f(x)=\dfrac{x-2}{x-3}$ $\quad$ $x-3\neq 0$, $x\neq 3$

$\boxed{\text{represent 'x' in terms of 'y'}}$

$\Rightarrow xy-3y=x-2$

$\Rightarrow xy-x=3y-2$

$x(y-1)=3y-2$

$\Rightarrow x=\dfrac{3y-2}{y-1\neq 0}$

$y-1\neq 0$

$y\neq 1$ $\quad$ $y\in R-\{1\}$

Range $= R-\{1\}$

Codomain $= R-\{1\}$

Range $=$ codomain

$\boxed{f(x)\rightarrow \text{onto}}$

$\boxed{Q.11}$ $f: R \to Ⓡ$ defined by $f(x) = x^4$

one-one / many-one

Let $f(x_1) = f(x_2) \Rightarrow$ $x_1 = x_2$ only

let $f(x_1) = f(x_2)$

$\Rightarrow x_1^4 = x_2^4$

$\Rightarrow x_1 = \pm x_2$

$\swarrow \quad \searrow$

$\boxed{x_1 = x_2}$ $\boxed{x_1 = -x_2}$

many-one

option Ⓓ

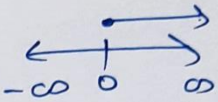
onto - into

Range ≠ Codomain $\to$ R

$x \in R$

$x^4 \geq 0$

$f(x) \geq 0$



$f(x) \in [0, \infty)$

Range $= [0, \infty)$

Not equal

Codomain $= R = (-\infty, \infty)$

not onto (into)

$\boxed{Q.12}$ $f: R \to Ⓡ$ defined by $f(x) = 3x$

one-one / many-one

Let $f(x_1) = f(x_2) \Rightarrow$ $x_1 = x_2$ only

let $f(x_1) = f(x_2)$

$3x_1 = 3x_2$

$\Rightarrow \boxed{x_1 = x_2}$

one-one ✓

$\boxed{\text{option-A}}$

onto - into

Range = Codomain $\to$ R ✓

$x \in R$

$3x \in R$

$f(x) \in R$

equal

Range $= R$ = Codomain

onto ✓

Composite Function & Invertible Function
(संयुक्त फलन) (प्रतिलोमीय फलन)

Note: Identity Function (तत्समक फलन) (I)

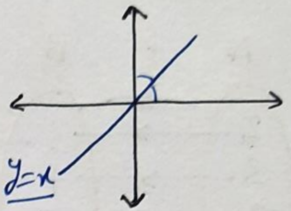
$f(x) = x$

$y = x$

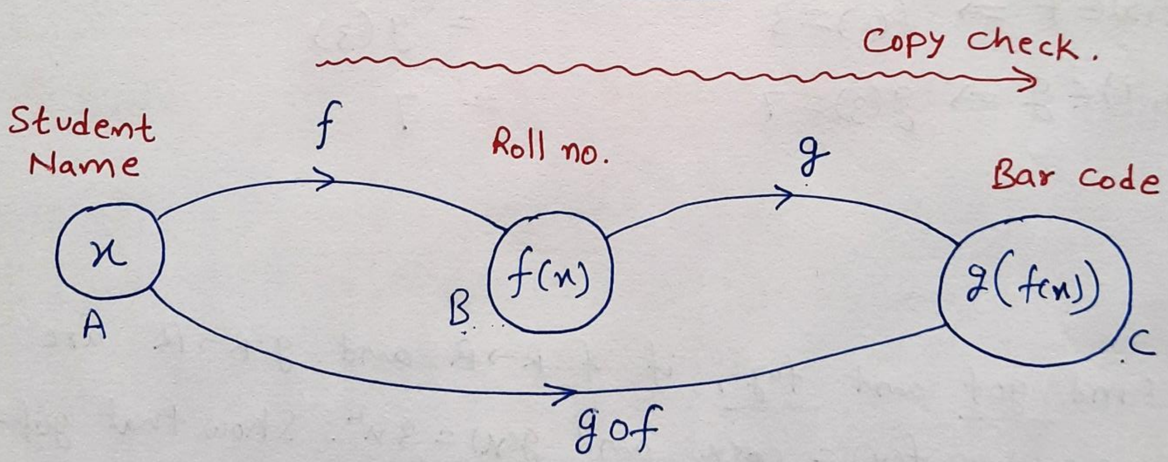
$f : R \to R$ given by $f(x) = x$ $(= I_R)$

$f : N \to N$ given by $f(x) = x$ $(= I_N)$

$f : A \to A$ given by $f(x) = x$ $(= I_A)$



Composite Function (संयुक्त फलन) ⇒



Definition: Let $f : A \to B$ and $g : B \to C$ be two functions. Then composition of 'f' and 'g' denoted by gof, is defined as the function $gof : A \to C$ given by

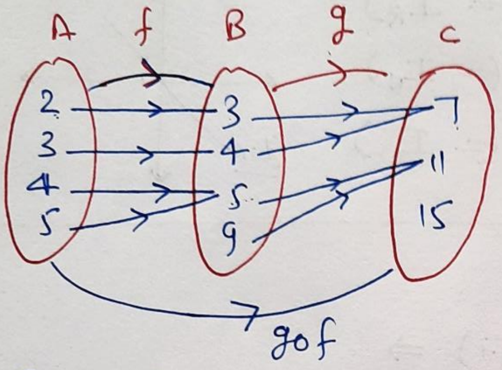
$$gof(x) = g(f(x)), \quad \forall\ x \in A$$

e.g. Let $f : \overset{A}{\{2,3,4,5\}} \rightarrow \overset{B}{\{3,4,5,9\}}$ and

$g : \underset{B}{\{3,4,5,9\}} \rightarrow \underset{C}{\{7,11,15\}}$ be defined as

$f = \{(2,3),\ (3,4),\ (4,5),\ (5,5)\}$ and

$g = \{(3,7),(4,7),\ (5,11),(9,11)\}$. Find $g o f$.



$$\underline{\underline{g}}\overset{B\rightarrow C}{g}\,o\,\overset{A\rightarrow B}{f} = \{(\underline{2,7}),(3,7),(4,11),(5,11)\}$$

$$\longleftrightarrow$$

$(2,3) \in f \Rightarrow f(2) = 3$

$(3,7) \in g \Rightarrow g(3) = 7$

$$gof(2) = g(\underline{\underline{f(2)}})$$
$$= g(3)$$
$$= 7$$

e.g. Find $\underline{gof}$ and $\underline{fog}$, if $f : R \rightarrow R$ and $g : R \rightarrow R$ are given by $\underline{f(x) = \cos x}$ and $\underline{g(x) = 3x^2}$. Show that $gof \neq fog$.

$$gof = gof(x)$$
$$= g(f(x))$$
$$= g(\cos x)$$
$$= 3(\cos x)^2$$
$$= 3\cos^2 x$$

$\boxed{x=0} \rightarrow \searrow \boxed{3}$

$$fog = fog(x)$$
$$= f(\underline{g(x)})$$
$$= f(3x^2)$$
$$= \cos(3x^2)$$

$\boxed{x=0} \rightarrow \searrow \boxed{1}$

$$\boxed{gof \neq fog}$$

Properties of composite Functions

① $g \circ f \neq f \circ g$ & $h \circ (g \circ f) = (h \circ g) \circ f$

② If f and g are one-one, then $g \circ f$ is $\boxed{\text{one-one}}$.

③ If f and g are onto, then $g \circ f$ is $\boxed{\text{onto}}$.

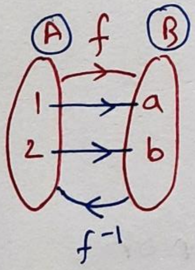
④ If $g \circ f$ is one-one, then 'f' is one-one.

⑤ If $g \circ f$ is onto, then 'g' is onto.

Invertible Functions (प्रतिलोमीय फलन) [f^{-1} / $f^{-1}(x)$]

Eligibility / योग्यता :→ Bijective Function

one-one & onto



let $f(x_1) = f(x_2)$
then $x_1 = x_2$ only

Range = Codomain
actually output
Question
$f: A \to B$

Definition : A function $f: X \to Y$ is defined to be invertible, if there exists a function $g: Y \to X$ such that $g \circ f = I_X$ and $f \circ g = I_Y$.

The function 'g' is called the inverse of f.

$$\boxed{g = f^{-1}}$$

$$f \circ f^{-1} = I = f^{-1} \circ f$$

$$\boxed{f \circ f^{-1}(x) = x = f^{-1} \circ f(x)}$$

Process to Find $\overset{f^{-1}(x)}{\uparrow}$ Inverse of a Function (f(x))

f(x) = expression

- check bijectivity of f(x).

$$\begin{tikzcd} & \text{bijectivity} \arrow[dl] \arrow[dr] & \\ \text{one-one} & & \text{onto} \end{tikzcd}$$

→ Let $y = f(x) = \boxed{\text{terms}}$ of x

→ write x in terms of y   ($x = \boxed{\text{terms}}$ of y)

→ Replace x by $f^{-1}(x)$

& y by x.

e.g. Find f^{-1} (if exists).

(i) $S = \{1, 2, 3\}$

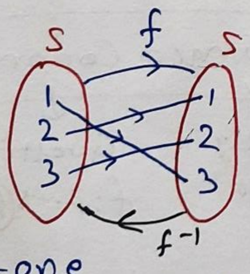
$f : S \to S$ given by

$f = \{(1,3), (3,2), (2,1)\}$

$f(1) = 3$

$f(3) = 2$

$f(2) = 1 \Rightarrow f^{-1}(1) = 2$



one-one ✓

onto ✓

Range = Codomain (S)

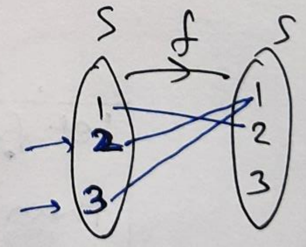
$S = \{1,2,3\}$

$f^{-1} = \{(3,1), (2,3), (1,2)\}$

(ii) $S = \{1,2,3\}$

$f : S \to S$ given by

$f = \{(1,2), (2,1), (3,1)\}$



one-one ✗ → not bijective → not invertible e.g. $f: \underline{R} \to R$ defined by $f(x) = 3x - 7$. Find f^{-1} if exists.

Bijectivity

one-one

$f(x_1) = f(x_2)$

$\Rightarrow 3x_1 - \cancel{7} = 3x_2 - \cancel{7}$

$\Rightarrow \boxed{x_1 = x_2}$ only

✓ one-one

onto

Range = Codomain

$f(x)\ R = (R)$

$x \in R$

$\Rightarrow 3x - 7 \in R$

$\Rightarrow f(x) \in R$

$\boxed{\text{Range} = R}$

onto ✓

$y = f(x) = 3x - 7$ (let)

$\Rightarrow y = 3x - 7$

$\Rightarrow \frac{y+7}{3} = x$

$\Rightarrow x = \frac{y+7}{3}$

then (after replacement)

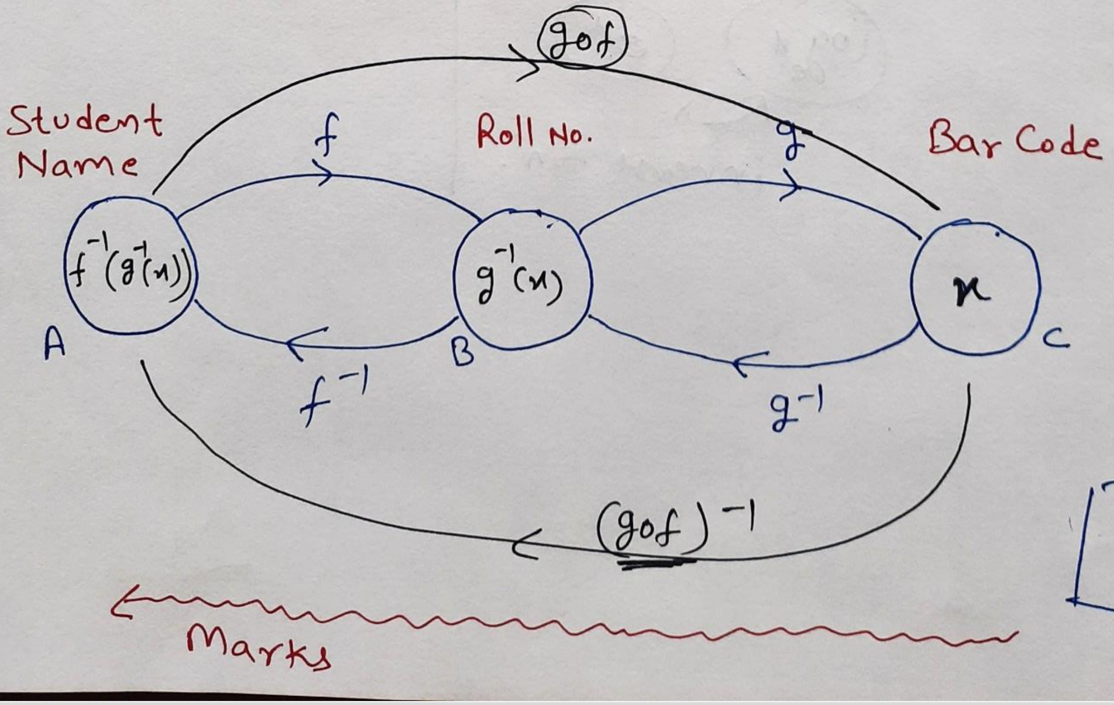
$\boxed{f^{-1}(x) = \frac{x+7}{3}}$

Properties of Invertible Functions

① $f \circ f^{-1}(x) = x = f^{-1} \circ f(x)$

② $(f^{-1})^{-1} = f$

☆③ $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ $\boxed{(g \circ f)^{-1}(x) = f^{-1} \circ g^{-1}(x)}$



$\frac{90}{100}$

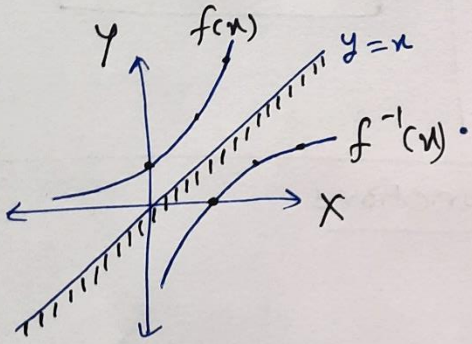
__Note__ : Let $f: X \to Y$ is a bijective function.

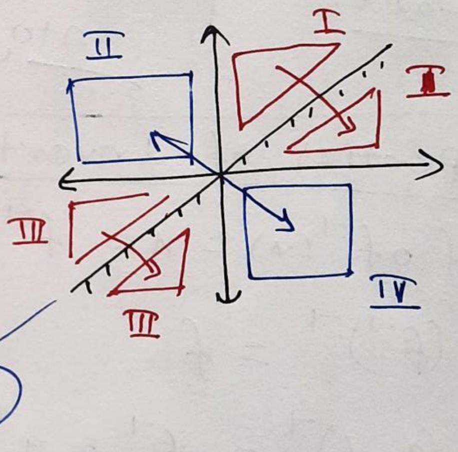
then $f^{-1}: Y \to X$ is a bijective function.

Domain of f = X = Range of f^{-1}

Range of f = Y = Domain of f^{-1}

__Graphically__, f and f^{-1} are mirror image of each other in __mirror line__ __$y=x$__.





$\log_e x$ e^x

inverse funⁿ.

[redacted]

Class 12 Maths (Rel^n & Fun^n.)

Exercise 1.3 (Composite Fun^n. & Inverse Fun^n.)

Q.1 $f:\overset{A}{\{1,3,4\}}\to\overset{B}{\{1,2,5\}}$, $g:\overset{B}{\{1,2,5\}}\to\overset{C}{\{1,3\}}$ $\quad$ gof = ?

$f=\{(1,2),(3,5),(4,1)\}$, $g=\{(1,3),(2,3),(5,1)\}$

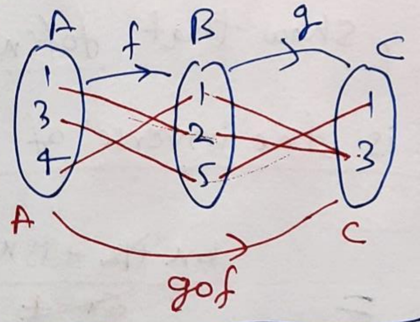
$f(1)=2$, $g(1)=3$
$f(3)=5$, $g(2)=3$
$f(4)=1$, $g(5)=1$

$$\boxed{gof=\{(1,3),(3,1),(4,3)\}}$$

$gof(1)=g(2)$
$=3$
$gof(3)=g(5)=1$

$gof(4)$
$=g(1)$
$=3$

$gof = gof(x)$ $\quad$ $f:A\to B$
x $\quad$ $f(x)$
$gof(x)$
C



$$\boxed{fog = fog(x) = f(g(x))}$$

Composite

Q.2 $f,g,h:R\to R$ $\quad$ Prove that: (i) $(f+g)oh = foh+goh$
(ii) $(f.g)oh=(foh).(goh)$

$(f\pm g)(x)=f(x)\pm g(x)$
$(f.g)(x)=f(x).g(x)$
$(f/g)(x)=\dfrac{f(x)}{g(x)}$
Algebra of Fn^n.
11th

(i) LHS $=(f+g)oh$
$=(f+g)oh(x)$
$=(f+g)(h(x))$
$=f(h(x))+g(h(x))$
$=foh+goh$
$=$ RHS

(ii) LHS $=(f.g)oh$
$=(f.g)(h(x))$
$=f(h(x)).g(h(x))$
$=foh.goh$
$=$ RHS.

③ Find gof and fog if

(i) $f(x) = |x|$ and $g(x) = |5x-2|$

$gof = g(f(x)) = g(|x|) = |5|x|-2|$ ✓

$||2|| = |2| = 2$

$||-2|| = |2| = 2$

$fog = f(g(x)) = f(|5x-2|) = ||5x-2|| = |5x-2|$

(ii) $f(x) = 8x^3$, $g(x) = x^{1/3}$

$gof = gof(x) = g(f(x)) = g(8x^3) = (8x^3)^{1/3} = 8^{\frac{1}{3}} \cdot (x^3)^{\frac{1}{3}}$

$= 2x$ ✓

$fog = f(g(x)) = f(x^{\frac{1}{3}}) = 8(x^{1/3})^3 = 8x$ ✓

---

④ If $f(x) = \dfrac{4x+3}{6x-4}$, $x \neq \dfrac{2}{3}$, show that $fof(x) = x$, for all $x \neq \dfrac{2}{3}$. What is the inverse of 'f'?

To Prove: $\boxed{fof(x) = x}$

LHS $= fof(x)$

$= f(f(x))$

$= f\left(\dfrac{4x+3}{6x-4}\right)$

$$fof(x) = \frac{4\left[\frac{4x+3}{6x-4}\right]+3}{6\left[\frac{4x+3}{6x-4}\right]-4}$$

$$= \frac{\dfrac{16x \cancel{+12} + 18x \cancel{-12}}{\cancel{6x-4}}}{\dfrac{\cancel{24x}+18-\cancel{24x}+16}{\cancel{6x-4}}}$$

$$= \frac{\cancel{34}x}{\cancel{34}} = x = \text{RHS}$$

$f: X \rightarrow Y$, $f^{-1}: Y \rightarrow X$

$fof^{-1}(x) = x = f^{-1}of(x)$

$fog(x) = x = gof(x)$

Let

$y = f(x) = \frac{4x+3}{6x-4}$

$\Rightarrow y = \frac{4x+3}{6x-4}$

$\Rightarrow 6\underline{x}y - 4y = 4\underline{x}+3$

$\Rightarrow 6\underline{x}y - 4\underline{x} = 4y+3$

$\Rightarrow x(6y-4) = 4y+3$

$\Rightarrow x = \frac{4\textcircled{y}+3}{6\textcircled{y}-4}$ $\Rightarrow \boxed{f^{-1}(x) = \frac{4x+3}{6x-4}} = f(x)$ ✓

① Let $y = f(x) = \_\_\_$

② x in terms of y

$x = \_\_\_(y)$

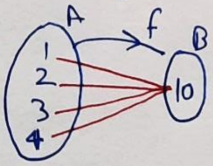
③ $x \rightarrow f^{-1}(x)$

$y \rightarrow x$

Q.5 Invertible $<$ Yes. / No. (with reason)

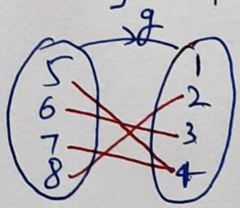
Bijective: one-one, onto

(i) $f: \overset{A}{\{1,2,3,4\}} \rightarrow \overset{B}{\{10\}}$, $f = \{(1,10),(2,10),(3,10),(4,10)\}$



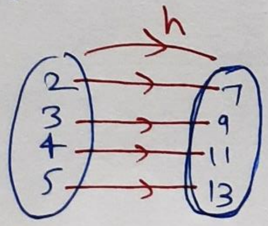
many-one   Not invertible

(ii) $g: \{5,6,7,8\} \rightarrow \{1,2,3,4\}$, $g = \{(\underline{5,4}), (6,3), (\underline{7,4}), (8,2)\}$



many-one & into   Not invertible

(iii) $h: \{2,3,4,5\} \rightarrow \{7,9,11,13\}$, $h = \{(2,7),(3,9),(4,11),(5,13)\}$



one-one, onto   Yes

Range = codomain

[redacted]

(12th maths) (Relations & Functions)

Exercise 1.3

Composite Fun. & Inverse Fun.

Q.6 Show that $f:[-1,1]\to \textcircled{R}$, given by $f(x)=\frac{x}{x+2}$ is one-one. Find the inverse of the function $f:[-1,1]\to$ Range of f.

Ans. $f(x)=\frac{x}{x+2}$

one-one   let $f(x_1)=f(x_2) \Rightarrow \boxed{x_1=x_2}$ only

$$\Rightarrow \frac{x_1}{x_1+2}=\frac{x_2}{x_2+2}$$

$$\Rightarrow \cancel{x_1x_2}+2x_1=\cancel{x_1x_2}+2x_2$$

$$\Rightarrow \boxed{x_1=x_2} \text{ only} \to f(x)\to \text{one-one,}$$

✓

Bijective → one-one ✓

Bijective → onto

Range = Codomain ✓

Codomain → Range of f

$f:A\to \textcircled{B}$

B: Codomain

Inverse of $f(x)=\frac{x}{x+2}$

Let $y=f(x)=\frac{x}{x+2}$

$\Rightarrow y=\frac{x}{x+2}$

$\Rightarrow xy+2y=x$

$\Rightarrow xy-x=-2y$

$\Rightarrow x(y-1)=-2y$

$\Rightarrow x=\frac{-2y}{y-1}$

$\Rightarrow f^{-1}(x)=\frac{-2x}{x-1}=\frac{2x}{1-x}$ ✓

① $y=f(x)$ (माना)

② x in terms of 'y'

③ Replace $x\to f^{-1}(x)$
$y\to x$

$\boxed{Q.7}$ Consider $f: \overset{\text{Domain}}{\underset{\downarrow}{R}} \to \textcircled{R}$ given by $f(x) = 4x+3$. Show that f is invertible. Find the inverse of f.

Bijective

one-one

let $f(x_1) = f(x_2)$

$\Rightarrow 4x_1 + \cancel{3} = 4x_2 + \cancel{3}$

$\Rightarrow \boxed{x_1 = x_2}$ only

one-one ✓

onto $\boxed{\text{Range} = \text{Codomain}}$

? (Actual output) $\qquad$ (R)

$x \to$ input $\quad x \in R$

$f(x) \to$ output

$x \in R$

$\Rightarrow 4x+3 \in R$

$\Rightarrow f(x) \in \textcircled{R}$ Range $= R =$ Codomain

onto ✓

$\therefore f$ is invertible,

$f(x) = 4x+3 = y$ (let)

$\Rightarrow 4x = y-3$

$\Rightarrow x = \dfrac{y-3}{4}$

Replace $x \to y \to f^{-1}(x)$

$y \to x$

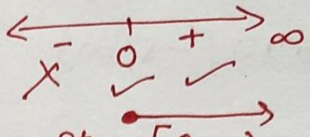
$\Rightarrow \boxed{f^{-1}(x) = \dfrac{x-3}{4}}$ ✓

$\boxed{gof(x) = x = fog(x) \quad g \to f^{-1}}$

$\boxed{Q.8}$ Consider $f: R_+ \to [4,\infty)$ given by $f(x) = x^2 + 4$.
Show that f is invertible with $f^{-1}(y) = \sqrt{y-4}$,
where R^+ is the set of all non-negative real no.

(x) Domain $= R_+ = [0,\infty) \to x_1, \to x_2$

Codomain $= [4,\infty)$



$R^+ = [0,\infty)$

Invertible → Bijective

one-one

$\boxed{\text{Let } f(x_1) = f(x_2) \Rightarrow \overset{\text{only}}{x_1 = x_2}}$

Let $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 + \cancel{4} = x_2^2 + \cancel{4}$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = \pm x_2$

$\boxed{x_1 = x_2}$ ✓

one-one

$\boxed{x_1 = -x_2}$ ✗

$x_2 \to R^+$

$x_1 \to \ominus$

✗

onto

$\boxed{\text{Range} = \text{Codomain}}$

Range → ?

Codomain → $[4,\infty)$

$f(x) \to x^2 + 4$

$\because x \in [0,\infty)$

$\Rightarrow x^2 \in [0,\infty)$

$\Rightarrow x^2 + 4 \in [4,\infty)$

$f(x) \in [4,\infty)$

Range $= [4,\infty) =$ Codomain

onto ✓

Let $y = f(x) = x^2 + 4$

$\Rightarrow y = x^2 + 4$

$\Rightarrow y - 4 = x^2$

$\Rightarrow x = \pm\sqrt{y-4}$

$\because x \in R^+ \Rightarrow x = \sqrt{y-4}$

Replacement.

$f^{-1}(x) = \sqrt{x-4}$

$f^{-1}(100) = \sqrt{100-4}$

$f^{-1}(y) = \sqrt{y-4}$ ✓

Q.9 $f : \boxed{R_+} \to [-5, \infty)$ $f(x) = 9x^2 + 6x - 5$

Show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6} - 1}{3}\right)$.

(x) Domain $= R_+ = [0, \infty)$

Codomain $= [-5, \infty)$

Invertible
↓
Bijective.

One-one

let $f(x_1) = f(x_2) \Rightarrow \boxed{x_1 = x_2}$ only

$\Rightarrow 9x_1^2 + 6x_1 - \cancel{5} = 9x_2^2 + 6x_2 - \cancel{5}$

$\Rightarrow 9(x_1^2 - x_2^2) + 6(x_1 - x_2) = 0$

$\Rightarrow 3(x_1 - x_2)[3(x_1 + x_2) + 2] = 0$

0 0

$\boxed{x_1 = x_2}$ only Accept

$3(x_1 + x_2) + 2 = 0$ ×

$\Rightarrow 3x_1 + 3x_2 + 2 = 0$

$\Rightarrow 3x_1 = -3x_2 - 2$

$\Rightarrow x_1 = \frac{-3x_2 - 2}{3}$

$x_2 \to +$
$x_1 \to -$

Reject

$x \in R_+$
$x_1 \in R_+$
$x_2 \in R_+$
Given.

One-one

Onto

Range = Codomain

? $[-5, \infty)$

$f(x) = 9x^2 + 6x - 5$ → Perfect Square.

$f(x) = 9x^2 + 6x - 5$

$= \underbrace{(3x)^2 + 2.(3x).1 + 1^2}{} - 1 - 5$

$f(x) = (3x + 1)^2 - 6$

$x \in [0, \infty)$

$\Rightarrow 3x \in [0, \infty)$

$\Rightarrow 3x + 1 \in [1, \infty)$

$\Rightarrow (3x + 1)^2 \in [1, \infty)$

$\Rightarrow (3x + 1)^2 - 6 \in [-5, \infty)$

$f(x) \in [-5, \infty)$

Range $= [-5, \infty) =$ Codomain

Onto

$$\underbrace{y = f(x) = 9x^2 + 6x - 5 = (3x+1)^2 - 6}$$

① $y = f(x)$

② x in terms of y

③ ~~$x \to f^{-1}(x)$~~

$$y = (3x+1)^2 - 6$$

$$\Rightarrow \quad y + 6 = (3x+1)^2$$

$$\Rightarrow \quad \pm\sqrt{y+6} = 3x+1$$

$$\Rightarrow \quad \pm\sqrt{y+6} - 1 = 3x$$

$$\Rightarrow \quad x = \frac{\pm\sqrt{y+6} - 1}{3}$$

$$\boxed{\begin{array}{c} y = f(x) \\ f^{-1}(y) = x \end{array}}$$

$\because x \in R^+$

$$x = \frac{\sqrt{y+6} - 1}{3} \Rightarrow \boxed{f^{-1}(y) = \frac{\sqrt{y+6} - 1}{3}}$$

Exercise 1.3 {Composite, Inverse Functions}

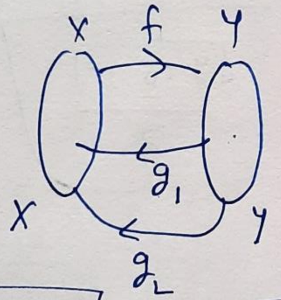
Q.10 Let $f: X \to Y$ be an invertible function. Show that f has unique inverse.

Proof: (Proof by Contradiction)

Let f does not have a unique inverse

let inverse of function f are g_1 & g_2.

$g_1 \neq g_2$.



f → invertible

Bijective

one-one → onto

$\boxed{f(x_1) = f(x_2) \Rightarrow x_1 = x_2}$

$\boxed{g_1 : Y \to X}$ also $\boxed{g_2 : Y \to X}$

$fog_1(x) = I_y = fo g_2(x)$ ← By Definition of Inverse.

$\Rightarrow fog_1(x) = fo g_2(x)$

$\Rightarrow f(g_1(x)) = f(g_2(x))$ ($\because f \to$ one-one)

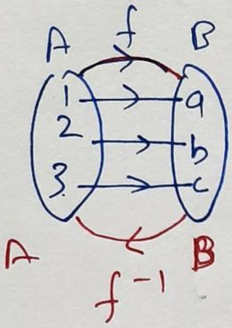
$\Rightarrow g_1(x) = g_2(x)$

$\Rightarrow g_1 = g_2$ This contradicts our assumption.

$\therefore$ f has a unique inverse.

Q.11 $f:\overset{A}{\{1,2,3\}}\to\overset{B}{\{a,b,c\}}$ $f(1)=a$, $f(2)=b$, $f(3)=c$.

Find f^{-1}. Show that $(f^{-1})^{-1}=f$



$f=\{(1,a),(2,b),(3,c)\}$

$\therefore f^{-1}=\{(a,1),(b,2),(c,3)\}=g$

$(f^{-1})^{-1}=\{(1,a),(2,b),(3,c)\}=g^{-1}$

$$\boxed{(f^{-1})^{-1}=f}$$

Q.12 Let $f:X\to Y$ be an invertible function. Show that inverse of f^{-1} is f i.e. $(f^{-1})^{-1}=f$.

Ans. $(f:X\to Y)$

Inverse of $f=f^{-1}=g$ (let) $\begin{pmatrix} f^{-1}:Y\to X \\ g:Y\to X \end{pmatrix}$

By definition,

$g o f=\overset{f(x)=x}{I}=f o g$ I → Identity Fn. $f(x)=x$

$g o f=I_X$ $f o g=I_Y$

→ $g o f=I_x$ → $f o g=x$

⇒ $g o f(x)=x$

$g o f=g o f(x)$

⇒ $f(x)=g^{-1}(x)$

$I:X\to X$

⇒ $f(x)=(f^{-1})^{-1}(x)$

⇒ $f=(f^{-1})^{-1}$

Q.13 If f: R → R be given by $f(x) = (3-x^3)^{\frac{1}{3}}$, then fof(x) is

(A) $x^{1/3}$ (B) x^3 [✓ (C) x] (D) $3-x^3$

Ans. $f(x) = (3-x^3)^{1/3}$

$$fof(x) = f(\underline{f(x)}) = f\left(\underbrace{(3-x^3)^{1/3}}\right)$$

$$= \left(3 - \left[(3-x^3)^{\cancel{1/3}}\right]^{\cancel{3}}\right)^{1/3}$$

$$= \left[3 - (3-x^3)\right]^{1/3} = (\cancel{3} - \cancel{3} + x^3)^{1/3}$$

$$fof(x) = x$$

Q.14 $f(x) = \dfrac{4x}{3x+4}$

Ⓐ $g(y) = \dfrac{3y}{3-4y}$ [✓ Ⓑ $g(y) = \dfrac{4y}{4-3y}$]

Ⓒ $g(y) = \dfrac{4y}{3-4y}$ Ⓓ $g(y) = \dfrac{3y}{4-3y}$

$\boxed{f^{-1} = g}$

① $y = f(x)$

② $x \to y$ ~~A~~ Terms

$$\Rightarrow y = f(x) = \frac{4x}{3x+4}$$

$$\Rightarrow y = \frac{4x}{3x+4}$$

$$\Rightarrow 3xy + 4y = 4x$$

$$\Rightarrow 4y = 4x - 3xy$$

$$\Rightarrow 4y = x.(4-3y)$$

$$\Rightarrow x = \frac{4y}{4-3y}$$

$$\Rightarrow f^{-1}(y) = \frac{4y}{4-3y}$$

$$\Rightarrow \boxed{g(y) = \frac{4y}{4-3y}}$$

$y = f(x)$

$f^{-1}(y) = x$

[redacted]

Class 12 Maths [Relations & Functions]

Binary Operations [द्विआधारी संक्रियाएं]

For State Boards

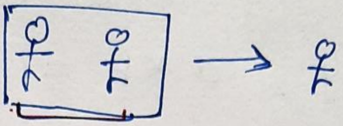
Binary → ② दो

⊕ ⊖ ⊗ ⊘

$\underbrace{\underbrace{100 + 35} + 98}$

~~++++~~

3 5 7



Set = Humans

Definition of Binary operations (*)   on A → from A to A

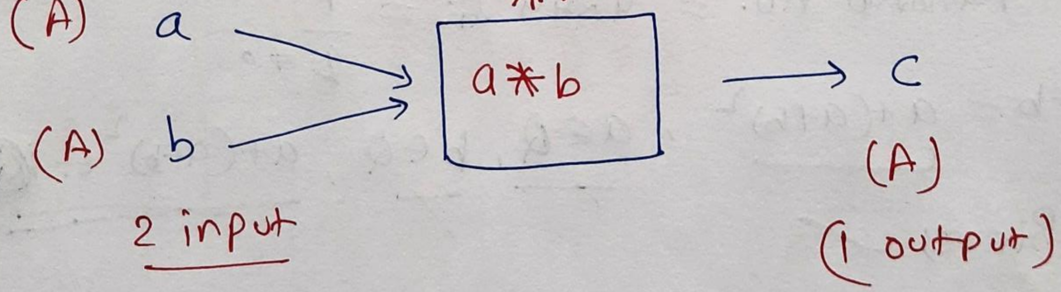
A binary operation '*' on a set A is a function * : $A \times A \to A$.

$\boxed{* : A \times A \to A}$

We denote $*(a,b)$ by $a * b$.

$*(a,b)$
↓
$f(a,b)$

$*: A \times A \to A$



e.g. $+(a,b)$ $a, b \in N$ (2,3) → $+(a,b) = +(2,3) = 2+3 = 5$

N N N

e.g. Check, which of the following $*$ is binary?

(i) On Z, $a*b = a-b$

(ii) on N, $a*b = a-b$

(iii) On Q, $a*b = a+(a+b)^2$

(iv) On $R-\{-1\}$, $a*b = \frac{a}{b+1}$

(i) $Z=$ integers $=\{\ldots -2, -1, 0, 1, 2 \ldots\}$

$a*b = a-b$ $\quad a\in Z, b\in Z \quad a-b \in Z$ ✓

$* \longrightarrow$ Binary ✓

(ii) $N=\{1,2,3,\ldots\}$ $\quad a\in N, b\in N, a-b \notin N$

$a*b = a-b$ $\quad a=1, b=5$

$1*5 = 1-5 = \boxed{-4} \notin N$ Not Binary

(iii) $Q=$ Rational no. $=$ परिमेय सं. $= \frac{p}{q\neq 0}$

$a*b = a+(a+b)^2$, $a\in Q$, $b\in Q$, $a+(a+b)^2 \in Q$

Binary ✓

IV $R-\{-1\}$

2 inputs $\quad$ 1 output

$a*b = \frac{a}{b+1}$

$a\in R-\{-1\}$ ✓

$b\in R-\{-1\}$ ✓

$\boxed{a=-\frac{1}{2}, b=-\frac{1}{2}}$

$\left(-\frac{1}{2}\right)*\left(-\frac{1}{2}\right) = \frac{-\frac{1}{2}}{-\frac{1}{2}+1} = \frac{\left(-\cancel{\frac{1}{2}}\right)}{\left(\cancel{\frac{1}{2}}\right)} = \boxed{-1}$

$\frac{a}{b+1} \notin R-\{-1\}$

Not Binary

Commutativity (क्रम विनिमेयता)

if $a * b = b * a$

Associativity (साहचर्यता) if $(a * b) * c = a * (b * c)$

Identity (तत्समक) $= e$ (identity of $*$)

$$a * e = \underset{\boxed{\uparrow}}{a} = e * a$$

Unique

Inverse (प्रतिलोम) $= b$ (माना)
(Let)

$$a * \underset{\downarrow}{b} = e = b * a$$

(inverse of a)

e.g. Consider a binary operation '$*$' on the set $\{1, 2, 3\}$ given by following operation table

b

a

$*$	1	2	3
1	1	2	3
2	2	2	3
3	3	3	3

$a * b$

(i) Compute $(2 * 3) * 1$
$= (3) * 1 = 3$

(ii) Compute $2 * (3 * 1)$
$= 2 * (3)$
$= 3$

(iii) Is $*$ commutative ?

$\boxed{a * b = b * 9}$

$2 * 3 = 3 * 2$ ✓
$1 * 3 = 3 * 1$ ✓
$1 * 2 = 2 * 1$ ✓

✓

e.g. Let $*$ be a binary operation on the set $\{1, 2, 3, 4, 5\}$ defined by

$$\boxed{a * b = \text{HCF of } a \;\&\; b}.$$

(i) Is $*$ a binary operations?

(ii) Is $*$ commutative?

(iii) find identity element for $*$?

Ans.

$$\underset{A}{\underset{\swarrow}{a}} * \underset{A}{\underset{\downarrow}{b}} = \underbrace{\text{HCF}(a, b)}_{A}$$

$A = \{1, 2, 3, 4, 5\}$

$1 * 2 = \text{HCF}(1, 2) = 1 \in A$

$2 * 4 = \text{HCF}(2, 4) = 2 \in A$

$3 * 5 = \text{HCF}(3, 5) = 1 \in A$

(i) Yes. (Binary ✓)

(ii) $a * b = \underline{\text{HCF of } a} \;\&\; b = \underline{\text{HCF of } b \;\&\; a}$

Yes <u>Commutative</u> $= b * a$

(iii) Identity Element $= e$

$\underbrace{a * e = a}_{} = \underline{e * a}$

$a \in A$

$a \in \{1, 2, 3, 4, 5\}$

$\boxed{a = 5}$

$5 * e = 5$

$\Rightarrow$ HCF of $5 \;\&\; e = 5$

$\boxed{e = 5}$

HCF 5 & 5

$\boxed{a = 4}$

$4 * e = 4$

$\Rightarrow$ HCF of $4 \;\&\; e = 4$

$\Rightarrow$ $\boxed{e = 4}$

$\boxed{e = 5 \quad e = 4}$

Not unique

Identity element does not exist